Numerical investigation of the effect of bed height on minimum fluidization velocity of cylindrical fluidized bed

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Abstract
Minimum fluidization velocity is one of the most important properties to characterize a bed, differentiating between a packed bed and a fluidized bed. Numerical simulation for 4 different ratios of initial bed heights (H) to base diameter (D), were performed; viz. 0.5, 1, 2 and 3. Glass beads of density 2600kg/m³ and with an average diameter of 550µm were used for all the simulations. Simulations were performed using the commercial CFD software, STAR-CCM+. The minimum fluidization velocity was identified by measuring pressure drop across the entire domain. The predicted minimum fluidization velocity remains constant for the different H/D ratios, proving that it is independent of bed height for a cylindrical bed.

1. Introduction

Fluidized beds have a wide range of application in the chemical, pharmaceutical, mineral and oil-gas industries. Manufacturing of polyethylene and polypropylene, the synthesis of various fuels,roasting and heat exchangers are some of the industrial application of fluidized beds. The reason for their widespread usage is the better mixing properties and the high contact surface area it provides between the 2 phases. This high contact area improves the efficiency of catalysts.

Depending on the type of the bubbles within the bed the flow is classified into different regimes-packed bed, bubbly flow, slug flow, churn flow and annular flow [2]. The bubbling regime occurs at moderate superficial velocities and contains small particles with very less transverse movement. There is no coalescence or break up of bubbles and the size of the bubbles formed is determined by the properties of fluid, particle and the distribution of the gas.

Several complexities are involved in numerical modeling of fluidized beds, the presence of gas-solid intermixing media-with a continuously changing interface, the transient nature and the interaction between the phases. This compounded nature of fluidized beds has been a hindrance in completely understanding the physics involved. With the advent of CFD, considerable progress has been made in conducting investigative studies relating to bed hydrodynamics. Two main numerical techniques have emerged in solving multi-phase problems, Eulerian model [3] [4] [5] [6] [7] [8] [9] and the Lagrangian model [10] [11] [12] [13] [14] [15].

The Eulerian model, assumes the 2 phases are continuous and inter penetrating. The general Navier-Stokes equation is solved for both the media, but additional closure laws are required to model the particle phase as continuous media. The inter phase interaction is accounted by the drag model and hence, utmost care has to be taken in choosing them. Studies relating to heat transfer [7], horizontal jet penetration [8] and particle rotation for segregation [5] has been conducted using the Eulerian model. The 2 methods have been compared by Gera et al, 1998 [16]. In the present study, Eulerian model is used for all the simulations.

A 2D Cartesian simulation is performed as opposed to a 3D cylindrical geometry, to save simulation time. 2D simulations must be used with caution and should be used only for sensitivity analysis, they predict the bed height and pressure drop with good accuracy but, for predicting the spatial position of particles it is preferable to use 3D simulations. Xie et al [6] have done extensive work in comparing results from 2D Cartesian, 2D axisymmetric and 3D calculations for bubbling, slugging and turbulent flow regime. Their results show that comparable results are obtained for all the cases of bubbling regime. Hence, it is justifiable to use 2D simulations for the present case.

Minimum fluidization velocity is one of the most important parameters to characterize a bed [17]. It is the velocity at which the weight of the bed is just balanced by the inertial force carried by the air coming into the bed. At
velocities just equal to or above minimum fluidization velocity the bed attains a suspended state. This velocity is a characteristic property because it depends on the particle property/geometry, bed geometry and fluid properties [18]. Gunn and Hilal [19] and Cranfield and Geldart [20] both showed that \( U_{mf} \) is independent of bed height for a certain types of beds like spouting beds and pseudo 2D beds.

2. Conditions in the actual experiment

The exact details about the experimental setup (Figure 1) and procedure are explained in the paper D.Escudero and T.J.Heindel, 2010 [1].

3. Computational model

The transport equations for momentum and continuity are solved for both the gas and the solid phase. The equations for the 2 phases are linked together through the drag law. The solid phase has additional equations solved for the kinetic, collisional and frictional regime fundamentally based on the kinetic theory of granular flow.

3.1. Continuity equation

The continuity equation for each phase is separately as shown:

\[
\frac{\partial}{\partial t} (\varepsilon_g \rho_g) + \nabla \cdot (\varepsilon_g \rho_g \mathbf{V}_g) = 0 \quad (1)
\]

\[
\frac{\partial}{\partial t} (\varepsilon_s \rho_s) + \nabla \cdot (\varepsilon_s \rho_s \mathbf{V}_s) = 0 \quad (2)
\]

The only constraint being that total volume fraction has to add up to one.

\[
\varepsilon_g + \varepsilon_s = 1. \quad (3)
\]

In the present work, there is no mass transfer between the phases and thus the terms on the right hand side of the equations 1 and 2 are zero.

3.2. Gas phase momentum equation

The virtual mass force and lift forces have been ignored in the following case. Virtual mass force plays a major role only in cases where the dispersed phase density is lower or comparable with the density of the
continuous phase. Lift force is concerned with the rotation of the dispersed phase, which is ignored in the present problem.

The gas phase momentum equation can be expressed as:
\[
\frac{\partial}{\partial t} (\epsilon_g \rho_g \vec{V}_g) + \nabla \cdot (\epsilon_g \rho_g \vec{V}_g \vec{V}_g) = \nabla \cdot \vec{r}_g - \epsilon_g \nabla P + \epsilon_g \rho_g g + \beta_{gs} (\vec{V}_g - \vec{V}_g) \tag{4}
\]
where \(P\) is the pressure, \(g\) is the acceleration due to gravity and \(\beta_{gs}\) is the drag coefficient (explained later). The stress tensor \(\vec{r}_g\) is calculated by the following equation:
\[
\vec{r}_g = \epsilon_g \mu_g \left(\nabla \vec{V}_g + (\nabla \vec{V}_g)^T\right) + \epsilon_g \left(\lambda_g + \frac{2}{3} \mu_g\right) \nabla \cdot \vec{V}_g \hat{I} \tag{5}
\]

3.3. Solid phase momentum equation

Assuming no virtual mass and lift force the solid phase momentum equation can be expressed as:
\[
\frac{\partial}{\partial t} (\epsilon_s \rho_s \vec{V}_s) + \nabla \cdot (\epsilon_s \rho_s \vec{V}_s \vec{V}_s) = \nabla \cdot \vec{r}_s - \nabla P_s - \epsilon_s \nabla P + \epsilon_s \rho_s g + \beta_{gs} (\vec{V}_s - \vec{V}_g) \tag{6}
\]
where \(P_s\) is the solid pressure obtained from the Kinetic theory of granular flow. This pressure has 3 components kinetic, collisional and frictional.

3.4. Kinetic theory of granular flow

Details of the model were first explained by Gidaspow [21] [22]. Assuming local dissipation of the granular energy, the granular temperature \(\theta_s\) is evaluated using an algebraic equation which account for the collisions between particles. The equation is expressed as:
\[
\theta_s = \left[\frac{-K_1 \varepsilon_s tr(\vec{t}_s) + K_2^{2} tr^{2}(\vec{t}_s) + 4K_4 \varepsilon_s \varepsilon_s [K_2 tr(\vec{t}_s) + 2K_3 tr(\vec{t}_s)]}{2\varepsilon_s K_4} \right]^2 \tag{7}
\]

\(tr(\vec{t}_s)\) is the stress tensor and the \(K\)'s are defined as:
\[
K_1 = 2(1 + e) \rho_s g_0 \tag{8}
\]
\[
K_2 = \frac{4}{3\sqrt{\pi}} d_s \rho_s (1 + e) \varepsilon_s g_0 - \frac{2}{3} K_3 \tag{9}
\]
\[
K_3 = \frac{d_s \rho_s}{2}\left[\frac{\sqrt{\pi}}{3(3-e)} \left[1 + \frac{5}{2} (1 + e)(3e - 1) \varepsilon_s g_0 \right] + \frac{8 \varepsilon_s}{5\sqrt{\pi}} g_0 (1 + e)\right] \tag{10}
\]
\[
K_4 = \frac{12(1-e^2) \rho_s g_0}{d_s \sqrt{\pi}} \tag{11}
\]
\[
P_k = \rho_s \varepsilon_s \theta_s \tag{12}
\]
\[
P_c = 2g_0 \rho_s \varepsilon_s^2 \theta_s (1 + e) \tag{13}
\]
\[
\mu_c = \frac{4}{5} \varepsilon_s \rho_s d_s (1 + e) \sqrt{\frac{\theta_s}{\pi}} \tag{14}
\]
\[
\mu_k = \frac{2 \mu_{dil}}{g_0(1+e)} \left[1 + \frac{4}{5} (1 + e) \varepsilon_s g_0 \right]^2 \tag{15}
\]
\[
\mu_{dil} = \frac{5\sqrt{\pi}}{96} (\varepsilon_s \rho_s) \left[\frac{d_s}{\varepsilon_s} \sqrt{\theta_s} \right] \tag{16}
\]

3.5. Schaeffer friction model

In regions where the contact between the particles is not instantaneous but continuous the friction between particles has to be considered. The model equations were originally described by Schaeffer [23] which
describe the plastic flow of a granular material and relate the shear stress to the normal stress. The Schaeffer model is only activated when the volume fraction of the particle exceeds a certain maximum packing limit (which is set as 0.65 in our case). The frictional pressure is modeled according to the following equation:

\[
P_f = \begin{cases} 
10^{25}(\varepsilon_s - \varepsilon_s^{\text{max}})^{10}, & \varepsilon_s > \varepsilon_s^{\text{max}} \\
0, & \varepsilon_s \leq \varepsilon_s^{\text{max}}
\end{cases} \quad (17)
\]

\[
\mu_f = \begin{cases} 
\min \left( \frac{P_f \sin(\phi)}{\sqrt{4I_{2D}}}, \mu_m^{\text{max}} \right), & \varepsilon_s > \varepsilon_s^{\text{max}} \\
0, & \varepsilon_s \leq \varepsilon_s^{\text{max}}
\end{cases} \quad (18)
\]

\[
I_{2D} = \frac{1}{6} \left[ (D_{s11} - D_{s22})^2 + (D_{s22} - D_{s33})^2 + (D_{s33} - D_{s11})^2 \right] + D_{s12}^2 + D_{s23}^2 + D_{s31}^2 \quad (20)
\]

The overall solid pressure is as solved as follows

\[
P_s = P_f + P_k + P_c \quad (22)
\]

The viscosity for the solid is modeled as

\[
\mu_s = \mu_f + \mu_k + \mu_c \quad (23)
\]

3.6. Drag models

Drag force is the most important force in fluidized beds as it is the only source of inter-phase interaction in fluidized beds. Some drag laws are obtained by experimental pressure drop data of packed beds. Ergun equation is one such mathematical model obtained for a packed bed. The Gidaspow drag model has a complementary Wen and Yu [24] model for lower volume fraction of particles (i.e., fluidized bed). Some details of Ergun and Wen & Yu equations are given in the paper by Robert K Niven [25]. The Gidaspow [26] model was used in current study and is formulated as follows.

\[
l_{gs} = \beta_{gs} (u_g - u_s) \quad (24)
\]

where \( \beta_{gs} \) is the inter phase drag coefficient and for the Gidaspow model given as follows:

\[
\beta_{gs} = \frac{3}{4} C_D \frac{\varepsilon_g \rho_g |u_g - u_s|}{d_s} \left( \varepsilon_g - 2.65 \right) \quad (25)
\]

\[
C_D = \begin{cases} 
\frac{24}{\varepsilon_g Re_s} \left[ 1 + 0.15 (\varepsilon_g Re_s)^{0.687} \right], & Re_s < 1000 \\
0.44, & Re_s > 1000
\end{cases} \quad (26)
\]

\[
\beta_{gs} = 150 \frac{\varepsilon_g \mu_g}{\varepsilon_g d_s^2} + 1.75 \frac{\varepsilon_g \rho_g |u_g - u_s|}{d_s} \quad \text{for} \ \varepsilon_g < 0.2 \quad (27)
\]

3.7. Turbulence

The continuous phase-Air, was modeled with k-ε turbulence and the dispersed phase-Glass beads, were modeled as laminar. The turbulence model for air was the standard k-ε model with RANS. Turbulent intensity, turbulent velocity ratio and turbulent viscosity ratio have been kept at default values available in STAR-CCM+. 
3.8. Solver and discretization scheme

The algorithm used to solve is SIMPLE. Second order discretization scheme was used for both velocity and volume fraction. Second order is preferred as it is more accurate compared to first order. However, in cases where the second order scheme was unstable, the velocity convection was changed to first order.

4. Numerical simulation

4.1. Mesh

Square cells were used in the domain with a refinement near the inlet to better capture the bed and the bubbles that form in the bed. The grid size and the refinement size near the inlet are given in Table 2. Similar mesh sizes have been used in previous work by Hosseini et al [27]. A mesh independence test was done by simulating a few cases with a refined mesh (33344 cells); the cell size in the entire domain was reduced by half.

4.2. Inlet distribution

Distributor plates play an important role in fluidized beds [28]. Various cases have been reported previously [29] [30] [31]. Complete sparging has been used in the present case to simulate the fluidized bed. Similar conditions have been used in previous research [27] [32] [33] [34] [35].

4.3. Initial conditions

The maximum packing fraction was set as 0.65, as explained earlier, and the initial packing fraction of the bed was chosen based on the bulk density reported in the experiment. The density of the particle is the same in all cases, and the ratio of mean bulk density-to-particle density gives the average volume fraction of particle in the bed, before starting the air flow. The bulk density values are reported in Table 1.

An initial velocity is given to the air which is equal to the superficial velocity divided by the volume fraction of air in the domain. This is done to give a good guess to the initial condition and so as to achieve quasi steady state quicker. Initial gauge pressure was set as 0 Pa throughout the domain.

4.4 Boundary conditions

The bottom boundary is set as velocity inlet, the top a pressure outlet and the 2 side walls are set as walls. The important parameter-superficial velocity is kept consistent with the required condition. Extrapolation condition was used for granular temperature on all the boundaries. This way we are not explicitly specifying the granular temperature to the particles but extrapolating to the boundary from the first layer of cells. The walls, on the sides, were given a no-slip condition for the fluid phase but a slip boundary condition for the particle phase i.e., the shear stress for the particle phase was set to 0. The top boundary was given a gauge pressure of 0 Pa so as to improve numerical accuracy of pressure drop. The volume fractions of particle and air were set to 0 and 1 respectively at the outlet.

4.5 Post processing

The pressure drop was measured by measuring the difference between the surface averaged pressure across the bottom (inlet) and the top boundary (outlet). The average of this is taken from 5s-15s with data acquisition at each time step. The averaging was started after 5 seconds of physical time as the bed achieves a quasi-steady state after approximately 5 seconds time.

The total mass was monitored by summing the product of volume fraction of particle in each cell, the volume of each cell and the density of the particle (2600kg/m³). This value obtained is the mass of the particles present in that “slice” of the experiment. Multiplying this by the factor to account for the 3D setup, we get the value as set from Table 1.
Pressure plot vs. time was plotted so as to see the trend as quasi-steady state is approached. The amplitude of oscillations was found to be increasing with increasing superficial velocities at superficial velocities greater than the minimum fluidization velocity; below minimum fluidization velocity the oscillations are negligible.

![Image](image.png)

**Figure 2. Mesh used in all simulations**

**Table 1. Bed material characteristics**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle density</td>
<td>2600kg/m³</td>
<td>Glass beads</td>
</tr>
<tr>
<td>Gas density</td>
<td>1.2kg/m³</td>
<td>Air at 20 deg C</td>
</tr>
<tr>
<td>Mean particle diameter(d)</td>
<td>550 µm</td>
<td>Uniform distribution</td>
</tr>
<tr>
<td>Coefficient of restitution(e)</td>
<td>0.9</td>
<td>Range in literature</td>
</tr>
<tr>
<td>Superficial gas velocity(U)</td>
<td>0.1m/s-0.3m/s</td>
<td></td>
</tr>
<tr>
<td>Bed width(D)</td>
<td>0.102m</td>
<td>Fixed value</td>
</tr>
<tr>
<td>Free board height</td>
<td>0.91m</td>
<td>~9D</td>
</tr>
<tr>
<td>Static bed height(H)</td>
<td>0.051m-0.306m</td>
<td>0.5D-3D</td>
</tr>
<tr>
<td>Grid spacing</td>
<td>0.005m</td>
<td>Specified</td>
</tr>
<tr>
<td>Grid refinement</td>
<td>0.0025m</td>
<td>Refinement of the bed</td>
</tr>
<tr>
<td>Time step</td>
<td>0.0001s</td>
<td>Specified</td>
</tr>
<tr>
<td>Cell count</td>
<td>8484</td>
<td>Fixed value</td>
</tr>
<tr>
<td>Maximum physical time</td>
<td>16s</td>
<td>Specified</td>
</tr>
</tbody>
</table>

**Table 2. Simulation parameters**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>H/D</td>
<td>Bed mass(g)</td>
<td>Bulk density(kg/m³)</td>
</tr>
<tr>
<td>0.5</td>
<td>670</td>
<td>1610±70</td>
</tr>
<tr>
<td>1</td>
<td>1320</td>
<td>1590±70</td>
</tr>
<tr>
<td>2</td>
<td>2560</td>
<td>1540±70</td>
</tr>
<tr>
<td>3</td>
<td>3610</td>
<td>1440±70</td>
</tr>
<tr>
<td>Diameter(µm)</td>
<td>500-600</td>
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</tr>
<tr>
<td>Particle Density(kg/m³)</td>
<td>2600</td>
<td></td>
</tr>
</tbody>
</table>

5. Results and discussion

The pressure drop across the bed increases with increase in H/D ratio; this is related to the increase in mass of the bed. On the other hand the minimum fluidization velocity (the velocity where the knee of the graph is obtained) is approximately same for the different H/D ratios. Hence, it can be concluded that there is no correlation between minimum fluidization velocity and bed height for cylindrical fluidized beds. The value of minimum fluidization velocity is approximately obtained to be at around 0.18m/s as shows in Figure 3. The exact value of minimum fluidization can only be obtained by performing more simulations near this value.
A force balance between the pressure drop and the weight of the bed is plotted as shown in Figure 4. The value of y axis is approximately one showing that beyond minimum fluidization the inertial force of the incoming air balances the weight of the bed.

The time history of pressure drop across the bed is shown in Figure 5. The pressure drop oscillates for velocities above minimum fluidization. Similar behavior was reported by Goldschmidt et al [9]. The reason for such behavior above minimum fluidization can be explained by assuming as the bed expands due to the incoming air, the resistance offered by the bed decreases and after a certain point this resistance is not enough to keep the suspended and the bed collapses back. This occurs repeatedly causing the oscillations.

The plot of experimental and simulation results of pressure drop are shown in Figure 6. The plots do not exactly coincide below the minimum fluidization velocity. A possible reason for this is the absence of wall friction and also, as reported by previous works, the Johnson and Jackson friction model works better than Schaeffer friction model.

Figure 7 shows the screen shot of a portion of the bed for different velocities. The bed is packed below minimum fluidization and just beyond that we start seeing disturbances and then with increasing superficial velocity we see bubbles forming within the bed. For Geldart-B type particles the minimum fluidization and minimum velocity for bubbling is expected to be the same though here we see that the bed gets fluidized at 0.18m/s but bubbles are seen only 0.21m/s and above. Further investigation is required to identify the reason.
6. Conclusion

Simulations were performed and the minimum fluidization velocity was determined to be independent of bed height for cylindrical beds. As discussed in literature, bed height affects minimum fluidization only in specialized
beds like 2D beds and spouted beds. The data obtained in this research corroborate with the data presented in the literature.

7. Notation

\begin{align*}
\varepsilon_s & \quad \text{Volume fraction of particle/solid} \\
\varepsilon_g & \quad \text{Volume fraction of air} \\
\rho_s & \quad \text{Density of particle} \\
\rho_g & \quad \text{Density of air} \\
\vec{V}_s & \quad \text{Velocity of solid (vector)} \\
\vec{V}_g & \quad \text{Velocity of gas (vector)} \\
\vec{\tau}_s & \quad \text{Stress tensor for solid} \\
\vec{\tau}_g & \quad \text{Stress tensor for gas} \\
\lambda_s & \quad \text{Bulk viscosity of gas} \\
P_s & \quad \text{Solid pressure} \\
P_f & \quad \text{Frictional pressure} \\
P_k & \quad \text{Kinetic pressure} \\
P_c & \quad \text{Collisional pressure} \\
\beta_{gs} & \quad \text{Drag coefficient} \\
g_o & \quad \text{Radial distribution function} \\
e & \quad \text{Coefficient of restitution} \\
d_s & \quad \text{Diameter of particle} \\
\varepsilon_{s, \text{max}} & \quad \text{Maximum volume fraction of particle} \\
\mu_{s, \text{max}} & \quad \text{Maximum viscosity of particle} \\
C_D & \quad \text{Standard drag coefficient} \\
Re_s & \quad \text{Particle Reynolds number} \\
d & \quad \text{Mean particle diameter} \\
U_{mf} & \quad \text{Minimum fluidization velocity} \\
u_s^{ad} & \quad \text{Advection velocity of solid} \\
u_g^{ad} & \quad \text{Advection velocity of gas} \\
l_{gs} & \quad \text{Drag force} \\
\mu_s & \quad \text{Solid phase viscosity} \\
\mu_f & \quad \text{Frictional viscosity} \\
\mu_k & \quad \text{Kinetic viscosity} \\
\mu_c & \quad \text{Collisional viscosity} \\
\mu_{dil} & \quad \text{Dilute viscosity}
\end{align*}

8. References

[1] David Escudero, Theodore J Heindel, Bed height and material density effects on minimum fluidization velocity in a cylindrical fluidized bed, 2010, 7th International Conference on Multiphase Flow, Tampa, FL, USA


[38] Esmali E, Mahinpey N, Adjustment of drag coefficient correlations in three dimensional CFD simulation of gas-solid bubbling fluidized bed, Advances in Eng. Sci., 2011, 375-386, 4