MODELLING OF BREAKUP AND COALESCENCE IN VERTICAL BUBBLY TWO-PHASE FLOWS

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ABSTRACT
Numerical simulations of gas-liquid two-phase flow with high superficial velocity in a vertical pipe were conducted with the use of the commercial software package STAR-CD 3.27. Detailed description of the $S_γ$ model is presented. The applicability and performance of the $S_γ$ model in Eulerian modelling of gas-liquid bubbly flow were studied. The sensitivity of the $S_γ$ model to the distribution moment $γ$, and the drainage mode were also investigated. The numerical results were compared with the available experimental data of Hibiki et al., (2001). Good agreement was achieved for the phase axial velocity and radial void fraction for all tested cases. It is found in this work that the second-moment $S_γ$ model in STAR-CD is capable of reasonably predicting bubble size and its distribution even in high void fraction. Except in the near wall region, simulated bubble size and therefore the interfacial area density does not fit well with the experiment measurements. It is observed that the predicted bubble size and interfacial area density obtained from both $S_0$ model and $S_2$ model are more or less the same, which indicates that the numerical results are independent of the distribution moment $γ$. It is further found that, the drainage mode greatly affects the bubble size: the increase of the mobility of the bubble surface enhances the coalescence in the current model and leads to an over-prediction of the bubble size in the pipe core. The bubble size increases with the increase of the gas phase superficial velocity while the interfacial area density varies less as the interfacial area density is a combined function of the bubble size and local gas hold-up.

Keywords: Breakup, bubbly flow, coalescence, Euler-Euler model, $S_γ$ model.

NOMENCLATURE

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Greek letters

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Subscripts

- $br$ breakup
- $c$ continuous phase
- $cl$ coalescence
- $coll$ collision
- $cr$ critical
- $d$ dispersed phase
- $eff$ effective
- $eq$ equivalent
- $i$ interfacial
- $k$ phase indicator
- $L$ laminar viscosity
- $L$ liquid phase
- $T$ shear-induced turbulent viscosity

INTRODUCTION

Bubble-driven flows occur widely in chemical, bio- and petro-chemical industrial processes. Proper
understanding of the fluid dynamics is essential to arrive at an optimum design and operation of such processes. Experimental investigation and numerical simulations are widely used to gain more knowledge and detailed physical understanding of this kind of complex gas-liquid flow processes.

Two approaches are mostly employed to simulate the bubble-driven flows: the Euler-Euler model (E-E) (Becker, Sololichin and Eigenberger, 1994) and Euler-Lagrange model (E-L) (Darmana, Deen and Kuipers, 2005). Detailed descriptions of the E-L and E-E models are easily found in the literature. It is commonly believed that the E-L method is more suited for the fundamental study of the bubbly flows while the E-E method is preferred in the high gas hold-up and churn turbulent flows. As this study deals with bubbly flow with high gas and liquid superficial velocities, E-E model is adopted here.

The hydrodynamics in bubble-driven flows are determined by the bubble rise and hence bubble size distribution and gas hold-up. In bubbly two-phase flows, the size of the bubble can undergo continuous change due to breakup and coalescence. Accurate description of the bubble size and its distribution are therefore of paramount importance in correctly simulating the two-phase flow behaviour. In the E-E model, the interfacial transfer terms are strongly related to the interfacial area concentration, $a_i$ and the local transfer mechanisms such as the degree of the turbulence near the interfaces. Though the MUSIG model (Lo, 1996) is capable of predicting a bubble size distribution, its computational effort and the assumption of the same slip velocity for all bubble size classes constrain its application to small scale geometries. Alternatively, during last decade, much attention (Wu et al., 1998; Hibiki and Ishii, 2000; Yao and Morel, 2004; Lo and Rao, 2007) has been concentrated towards developing an interfacial area concentration transport equation to describe the temporal and spatial evolution of the two-phase interface structure. The main difference among the aforementioned interfacial area concentration transport equation models is the closures for the breakup and coalescence, and more details were presented in the work of Yeoh and Tu (2004). Among these models, the volumetric conserved $S_\gamma$ model is based on a pre-assumed a log-normal bubble size distribution. With $\gamma$ to represent different moments of the distribution, researchers have more choices to solve for: bubble number density ($\gamma = 0$), interfacial area density ($\gamma = 2$), and/or volume fraction ($\gamma = 3$). Hence $S_\gamma$ model is investigated in this study.

In this work, two-dimensional simulations of gas-liquid bubbly flows were performed with the use of an Euler-Euler model for the cases of vertical pipe flows. Detailed description of the $S_\gamma$ model in the STAR-CD software, which was developed to model the size distribution of dispersed bubbles or droplets with the consideration of breakup and coalescence effects, is also presented. All the numerical results are compared with the experimental data of Hibiki et al., (2000).

**TWO-FLUID MODEL DESCRIPTION**

The STAR-CD Eulerian two-phase flow model solves the conservation of mass, momentum, and energy of the two phases in each cell. Full details of the Eulerian two-phase flow models in STAR-CD can be found in the work of Lo (2005) or STAR-CD manual (2004). The generic conservation equations for mass and momentum respectively take the following form:

\[
\frac{\partial (\alpha_i \rho_i)}{\partial t} + \nabla \cdot \left( \alpha_i \rho_i \mathbf{u}_i \right) = 0 \tag{1}
\]

\[
\frac{\partial (\alpha_i \rho_i \mathbf{u}_i)}{\partial t} + \nabla \cdot \left( \alpha_i \rho_i \mathbf{u}_i \mathbf{u}_i + \alpha_i \mathbf{r}_i \right) = \mathbf{a}_i \rho_i \mathbf{g} - \mathbf{a}_i \nabla p_i + M_i \tag{2}
\]

where the index $k$ refers to the phase under consideration ($L$ for liquid, $G$ for gas). $\mathbf{u} = (u,v,w)$ is the velocity vector. The volume fraction of each phase is denoted by $\alpha$. $M_i$ represents the inter-phase momentum exchange between phase $k$ and all other phases, and accounts for the interface forces. In this study, drag, turbulent dispersion, virtual mass and lift forces are accounted for. Full details of these forces can be found in Lo (2005) and STAR-CD Manual (2004).

For phase $k$, the stress tensor $\mathbf{r}_i$ appearing in Eq.2 reads:

\[
\mathbf{r}_i = -\mu_{k,eff} \left( \nabla \mathbf{u}_i + (\nabla \mathbf{u}_i)^T - \frac{2}{3} \nabla \nabla \cdot \mathbf{u}_i \right) \tag{3}
\]

$\mu_{k,eff}$, for the liquid phase ($k = L$) is composed of the molecular viscosity $\mu_{L,L}$ and the shear-induced turbulent viscosity $\mu_{L,T}$:

\[
\mu_{L,eff} = \mu_{L,L} + \mu_{L,T} \tag{4}
\]

The liquid phase shear-induced turbulent viscosity is evaluated by the $k - \varepsilon$ turbulence model and it is given by:

\[
\mu_{L,T} = C_p \rho_L \frac{k_L^2}{\varepsilon_L} \tag{5}
\]

The conservation equations for the extended $k - \varepsilon$ turbulence model are respectively given by:

\[
\frac{\partial (\alpha_L k_L)}{\partial t} + \nabla \cdot \left( \alpha_L \rho_L \mathbf{u}_L k_L - \alpha L \mu_{L,L} \frac{\mathbf{r}_L}{\sigma_k} + \frac{\mathbf{r}_L \cdot \nabla k_L}{\sigma_k} \right) = \alpha_L (G - \rho_L c_L) + S_{k_2} \tag{6}
\]

\[
\frac{\partial (\alpha_L \varepsilon_L)}{\partial t} + \nabla \cdot \left( \alpha_L \rho_L \mathbf{u}_L \varepsilon_L - \alpha L \mu_{L,L} \frac{\mathbf{r}_L}{\sigma_\varepsilon} + \frac{\mathbf{r}_L \cdot \nabla \varepsilon_L}{\sigma_\varepsilon} \right) = \alpha_L \frac{c_L}{k_L} (c_L G - \rho_L c_L) + S_{\varepsilon_2} \tag{7}
\]

where

\[
G = \mu_{L,T} \left( \nabla \mathbf{u}_L + \nabla \mathbf{u}_L^T \right) : \nabla \mathbf{u}_L \tag{8}
\]
The second-moment of the distribution, $S_2$, is related to the volume fraction of the bubbles $\alpha$

$$
S_2 = \frac{3}{4} \frac{C_p}{d_g} \alpha \rho_L |u_g - u_L| (C_i - 1) k_L 
\begin{equation}
(9)
\end{equation}
$$

$$
S_2 = \frac{3}{2} \frac{C_p}{d_g} \alpha \rho_L |u_g - u_L| (C_i - 1) \epsilon_L 
\begin{equation}
(10)
\end{equation}
$$

The gas phase ($k = G$) stress tensor is calculated as follows:

$$
\tau_G = \frac{\rho_g}{\rho_L} C_i^2 \tau_L 
\begin{equation}
(11)
\end{equation}
$$

where $C_i$ is a response coefficient defined as the ratio of the dispersed phase velocity fluctuations to those of the continuous phase:

$$
C_i = \frac{u'_g}{u'_L} 
\begin{equation}
(12)
\end{equation}
$$

Further details of the response coefficient $C_i$ and the turbulence model can be found in the work of Lo (2005) and the STAR-CD manual (2004).

**MODEL DESCRIPTION**

The evolution of the bubble size distribution as a result of coalescence and breakup can be modelled by means of the population balance equation (PBE). A detailed description of the bubble size distribution will require a large number of population classes, say 10 to 20. The computational effort required to solve 10 to 20 PBE’s simultaneously in a CFD calculation can be large. A simpler alternative can be derived by assuming that the bubble size distribution conforms to a pre-defined shape, and this shape is retained during the process under investigation. Under these assumptions, the complete bubble size distribution can be represented by a limited number of parameters and the PBE could be reformulated in terms of these parameters. $S_3$ is conserved on a volumetric basis and is related to the moment $M_3$ of the distribution:

$$
S_3 = n M_3 = \int_0^n d^3 P(d) d(d) 
\begin{equation}
(13)
\end{equation}
$$

where $n$ is the number density of the bubbles, $n = S_0$, i.e. the zeroth-moment of the distribution is the bubble number density.

The second-moment of the distribution, $S_2$, is related to the interfacial area density $a_i$:

$$
a_i = \int \pi d^2 P(d) d(d) = \pi S_2 
\begin{equation}
(14)
\end{equation}
$$

where $n$ is the number density of the bubbles, $n = S_0$, i.e. the zeroth-moment of the distribution is the bubble number density.

The third-moment of the distribution, $S_3$, is related to the volume fraction of the bubbles $\alpha$

$$
\alpha = \frac{\int \pi d^3 P(d) d(d)}{6} = \frac{\pi S_3}{6} 
\begin{equation}
(15)
\end{equation}
$$

The Sauter Mean diameter can be obtained:

$$
d_{32} = \frac{S_1}{S_2} = \frac{6 \alpha}{\pi \epsilon} 
\begin{equation}
(16)
\end{equation}
$$

Or

$$
d_{50} = \left( \frac{S_1}{S_0} \right)^{\frac{1}{3}} = \left( \frac{6 \alpha}{\pi} \frac{1}{S_0} \right)^{\frac{1}{3}} 
\begin{equation}
(17)
\end{equation}
$$

The transport equation for $S_3$ is given by:

$$
\frac{\partial S_3}{\partial t} + \nabla \cdot (S_3 \mathbf{u}_G) = s_{br} + s_{cl} 
\begin{equation}
(18)
\end{equation}
$$

where $s_{br}$ and $s_{cl}$ are the source terms for breakup and coalescence respectively.

**Breakup model**

Breakup will occur only if the bubble is larger than the critical diameter, $d_{cr}$, i.e., the so-called maximum stable bubble diameter. Viscous breakup is found in laminar flows and in turbulent flows for bubble smaller than the Kolmogorov length scale. Larger bubbles are subjected to inertial breakup. The Kolmogorov length scale $L_k$ is given by:

$$
L_k = \left( \frac{\nu^3}{\epsilon} \right)^{\frac{1}{4}} 
\begin{equation}
(19)
\end{equation}
$$

where $\nu$ is the continuous phase kinematic viscosity and $\epsilon$ is the continuous phase dissipation rate of turbulent kinetic energy.

In the case of turbulent flow with $d_{cr} < L_k$ the breakup source term is given by the sum of the sources for the viscous ($d_{cr} < d < L_k$) and inertia ($d > L_k$) regimes:

$$
S_{br} = s_{br,v} + s_{br,i} 
\begin{equation}
(20)
\end{equation}
$$

The breakup source term in its generic form is given by:

$$
s_{br} = \int_0^d d^3 \left( N_f(d) \frac{\gamma \eta}{\tau_{br}(d)} \right) n P(d) d(d) 
\begin{equation}
(21)
\end{equation}
$$

The reciprocal of the breakup time, $1/\tau_{br}$, represents the breakup rate $K_{br}$, $N_f$ is the daughter bubble number of diameter $d$, resulting from the breakup of a bubble of diameter $d$. In the current implementation of the model, only binary breakup is considered in which bubbles are broken into two fragments of equal size. Therefore:
\[
N_f(d) = 2 \quad (22)
\]

**Viscous breakup**

The breakup criterion follows from a balance between disruptive and restoring forces: the viscous stress and Laplace pressure respectively. This force balance is expressed in terms of the capillary number, \( \Omega \):

\[
\Omega = \frac{\mu_\ell d_s^2 \gamma}{2\sigma} \quad (23)
\]

where \( \mu_\ell \) is the dynamic viscosity of the continuous phase, \( \sigma \) is the surface tension coefficient and \( \gamma \) is the shear rate of the continuous phase. For laminar flows, the shear rate, \( \dot{\gamma} \) is calculated from the local velocity gradient; for turbulent flows the Kolmogorov shear rate is used:

\[
\dot{\gamma} = \sqrt{\frac{2\epsilon}{\mu_\ell}} \quad (24)
\]

where \( \rho_\ell \) is the density of the continuous phase.

According to De Bruijn (1987), the breakup condition is a function of the viscosity ratio \( \lambda = \mu_\ell/\mu_s \) and the flow type (laminar or turbulent). The breakup criterion is given as \( \Omega \geq \Omega_c \). The critical diameter is therefore given by:

\[
d_{cr} = \frac{2\sigma\Omega_c}{\mu_\ell \dot{\gamma}} \quad (25)
\]

Based on the dimensional analysis it can be derived that the breakup time, \( \tau_{br} \), in the viscous breakup regime takes the following form:

\[
\tau_{br} = \frac{\mu_\ell d_s^3}{\sigma} f_s(\lambda) \quad (26)
\]

where the function \( f_s(\lambda) \) has been correlated to the experimental data by Grace (1982) for viscous flows:

\[
\log f_s(\lambda) = p_0 + p_1 \log(\lambda) + p_2 (\log(\lambda))^3 \quad (27)
\]

the values of \( p_0, p_1 \) and \( p_2 \) have been determined from experimental data and can be found in the work of Lo (2005).

**Inertia breakup**

Inertia breakup is found in turbulent flows for bubbles larger than the Kolmogorov length scale, \( L_k \). The breakup criterion is formulated in terms of the dimensionless Weber number:

\[
We = \frac{\rho_s \dot{\gamma}^2 d_s^{5/3}}{2\sigma} \quad (28)
\]

Breakup occurs when \( We \geq We_{cr} \), where \( We_{cr} \) is a function of \( Re_{cr} \), the Reynolds number based on the critical bubble diameter. Furthermore the presence of nearby bubbles dampens the disruptive power of the inertia forces, a correction factor containing the volume fraction of bubble is used in the calculation of the critical bubble diameter:

\[
d_{cr} = (1 + C_a) \left( \frac{2\sigma We_{cr}}{\rho_c} \right)^{3/5} e^{-2/5} \quad (29)
\]

where \( C_a \) is the dispersed phase concentration correction factor and takes a value of 4.6. Based on the work of Yao and Morel (2001), the value of \( We_{cr} = 0.31 \) was used in this study.

In the inertia breakup regime, the breakup time scale follows from the frequency of the second oscillation mode of the droplet:

\[
\tau_{br} = 2\pi k_{br} \sqrt{\frac{3\rho_p + 2\rho_s}{192\sigma}} d^{3/5} \quad (30)
\]

where \( k_{br} = 0.2 \) is the inertia breakup time constant and \( \rho_s \) is density of the dispersed phase.

**Coalescence model**

When two bubbles collide they interact for a certain amount of time, forming a dumbbell. During this interaction the film of the continuous phase between the bubbles will start to drain. If drainage proceeds down to a certain critical film thickness \( h_{cr} \) within the provided interaction time, coalescence will take place; otherwise, the bubbles will separate.

The generic form of the coalescence source term is:

\[
s_{eq} = \int_0^\infty \int_0^\infty K_{cl} d_{cl} \Delta S_{cl} d_{cl} n^2 P(d') dd' P(d) dd \quad (31)
\]

The change in \( S_t \) due to a single coalescence event, \( \Delta S_{cl} \), can be deduced from the condition of volume conservation during coalescence. Consider two bubbles with diameters \( d \) and \( d' \), coalescing together to form a bubble with diameter \( d'' \):

\[
\Delta S_{cl} = d'' \left( 1 + \frac{d'}{d} \right)^{1/3} - (1 + \frac{d'}{d}) \quad (32)
\]

Assume that the bubble size has a uniform distribution with an equivalent mean diameter, \( d_{eq} \). For an arbitrary function of \( d_{eq}(d_{eq}) \), we have:

\[
\int_0^n f(d_{eq}) nP(d) dd = n_{eq} f(d_{eq}) \quad (33)
\]

where \( n_{eq} = \frac{6a_s}{\pi d_{eq}^3} \).

The equivalent diameter \( d_{eq} \) is chosen to be proportional to \( d_{cl} \):

\[
d_{eq}(\gamma) = k_{cl} d_{cl} \quad (34)
\]

Always, the proportionality constant, \( k_{cl} \) is of order unity. Then Eq. 32 is simplified to:
\[ \Delta S_{cl}^{d,d'} = d_{eq}^{2/3} \left(2^{1/3} - 2\right) \]  

(35)

The coalescence rate \( K_{cl} \) is given by:

\[ K_{cl} = F_{cl} k_{coll} d_{eq}^2 u_{rel} P_{cl}(d_{eq}) \]  

(36)

where \( F_{cl} \) is the calibration coefficient; \( k_{coll} \) is the collision rate coefficient, and \( P_{cl} \) is the coalescence probability of a single collision event. \( u_{rel} \) is the typical velocity difference over a range of \( d \). It should be noted that \( k_{coll}, P_{cl} \) and \( u_{rel} \) are determined by the collision regime.

Based on Eqs. 35 and 36, Eq. 31 is re-arranged as:

\[ s_{cl} = F_{cl} \left(2^{1/3} - 2\right) \frac{6\rho}{\pi} k_{coll} u_{rel} P_{cl}(d_{eq}) d_{eq}^{-3} \]  

(37)

Viscous collision

For viscous collisions:

\[ k_{coll} = \left(\frac{8\pi}{3}\right)^{1/3} \]  

(38)

\[ u_{rel} = \dot{\gamma} d \]  

(39)

For the viscous collision, the coalescence probability, \( P_{cl} \) is linked to the ratio of the interaction time, \( t_{i} \) and the film drainage time, \( t_{d} \):

\[ P_{cl} = \exp \left(-\frac{t_{i}}{t_{d}}\right) \]  

(40)

The interaction time scale is given by:

\[ t_{i} = \frac{1}{\dot{\gamma}} \]  

(41)

The drainage time, \( t_{d} \) depends on the mobility of the interface. For this study, we considered three different drainage modes, indicating the extent of the bubble surface mobility. As drainage mode 1 is valid for the droplets with fully immobile interface, we start from drainage mode 2.

Drainage mode 2:

\[ t_{d} = \frac{3}{2} \left(\frac{F_{cl} d^{2} \sqrt{\mu_{d} \rho_{d}}}{32\pi \sigma^{3} h_{cr}}\right)^{2/3} \]  

(42)

Drainage mode 3:

\[ t_{d} = \frac{\pi \mu_{d} \sqrt{\mu_{d} \rho_{d}}}{2 h_{cr}} \left(\frac{d_{eq}}{4\pi \sigma}\right)^{3/2} \]  

(43)

Drainage mode 4:

\[ t_{d} = \frac{3 \mu_{d} d_{eq}}{4 \sigma} \ln \left(\frac{d_{eq}}{8h_{cr}}\right) \]  

(44)

The interaction force during the collision is given by:

\[ F_{i} = \frac{3\pi}{2} \mu_{d} \dot{\gamma} d^{2} \]  

(45)

and the critical film thickness is obtained from:

\[ h_{cr} = \left(\frac{A_{H} d}{24\pi \sigma}\right)^{1/3} \]  

(46)

where \( A_{H} = 5 \times 10^{-21} \) is the Hamaker constant.

Inertia collision

For inertia collisions:

\[ k_{coll} = \frac{2\pi}{15} \]  

(47)

\[ u_{rel} = \left(\frac{c_{e} d_{eq}}{d_{eq}}\right)^{3} \]  

(48)

During inertia collisions, bubble shape oscillations may have a dominant influence on the local approach velocity in the film. In particular the phase difference between the oscillating bubbles determines the (local) time of contact, and hence the coalescence probability. Chester (1988) presents an expression for the coalescence probability in such cases:

\[ P_{cl} = \frac{\Phi_{max}}{\pi} \left(1 - \left(\frac{k_{d,2}^{2} (We_{cr} - We_{0})}{\Phi_{max}^{2}}\right)^{1/2}\right) \]  

(49)

\[ u_{rel} = \left(\frac{e_{c} d_{eq}}{d_{eq}}\right)^{3} \]  

(50)

where \( k_{d,2} \approx 12.7, We_{0} = 0.8 We_{cr} \) and \( h_{0} = 8.3 h_{cr} \) and \( \Phi_{max} \) is the maximum phase difference and is given by:

\[ \Phi_{max} = \frac{2k_{d,2}^{2} \rho_{d} \sigma}{We_{cr} \mu_{d}^{2} d} \]  

(51)

PHYSICAL PROBLEM

The experimental data sets published by Hibiki et al. (2001) provide measured data on void fraction, interfacial area density, bubble diameter, gas and liquid velocities for bubbly flows in a vertical pipe. A wide range of gas and liquid flow rates were studied. These data sets therefore provide very valuable information for checking and validating the \( S_{f} \) model for gas-liquid flows.

The test section of Hibiki’s experiments is a vertically placed circular pipe. Inner diameter of the pipe is 50.8 mm and the height is 3.06 m. Water and air at atmospheric condition were used. Measurements were taken in two axial positions: \( z/D = 6.0 \) and \( z/D = 53.5 \), radial position varies from \( r/R = 0 \) to \( r/R = 0.95 \). Hibiki et al. (2001) reported that the initial bubble diameter was about 3 mm at the pipe inlet. Dispersed air bubbles and water were injected into the pipe through the bottom plane.
Table 1: Case definition and parameters used.

<table>
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<tr>
<th>Case</th>
<th>$j_G$ (m/s)</th>
<th>$j_L$ (m/s)</th>
<th>$\gamma$</th>
<th>Drainage mode</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.986</td>
<td>2</td>
<td>3</td>
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<td>2.01</td>
<td>2</td>
<td>3</td>
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<tr>
<td>6</td>
<td>0.624</td>
<td>2.01</td>
<td>2</td>
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</tr>
</tbody>
</table>

$C_D = 1.071, C_L = -0.288, C_{VM} = 0.5$

$\rho_G = 1000 \text{ kg/m}^3, \rho_L = 1.29 \text{ kg/m}^3, \sigma = 0.07275 \text{ N/m},$

$\mu_{L,Lam} = 0.001 \text{ kg/(m.s)}, \mu_{G,Lam} = 1.812 \times 10^{-5} \text{ kg/(m.s)}.$

$\delta_{B,ini} = 3 \text{ mm}$

### NUMERICAL SOLUTION METHOD

All the numerical simulations are carried out with the commercial CFD package STAR-CD 3.27. The flow domain is two-dimensional axisymmetric and it is subdivided by 3000 grid cells (20 in radial and 150 in axial directions). In this work, the bubbly flow is treated as steady state. Second-moment $S_\gamma$ model with breakup and coalescence was solved to capture the evolution of the bubble size. The convergence criteria is all residuals below $1 \times 10^{-6}$. A summary of the simulation parameters and physical properties used are presented in Table 1.

### RESULTS AND DISCUSSION

The applicability and performance of the $S_\gamma$ model to predict the bubble-size distribution in the frame of the E-E model are studied with the help of the data sets of Hibiki et al. (2001). The employed experiments are listed in Table 1. The applicability of the model is investigated first; subsequently, the sensitivity of the $S_\gamma$ model to the distribution moment, $\gamma$ and the drainage mode in the viscous coalescence are evaluated. Finally, the performance of the model with the best model constants is assessed for the experimental data sets.

**The applicability of the $S_\gamma$ model**

Figure 1 presents the simulated bubble size distribution of test case 1. First of all, it is observed in Figure 1 that the bubble size ranges from 1 to 5 mm, which is in reasonable agreement with the experimental data of Hibiki et al., (2001), in which, the bubble size varied from 2 to 5 mm in the test case. It is clearly shown in this figure that the bubble size increases with the increasing of the distance from the sparger. This finding is also consistent with the experimental results and is due to the coalescence occurring inside the column. Furthermore, it is found in Figure 1 that bubble size near the wall is smaller. There are two reasons for this: as observed in Figure 2, the gas phase volume fraction profile exhibits a core peaking shape, the volume fraction near the wall is smaller, subsequently, the collision probability is reduced, which weakens the coalescence; furthermore, as can be easily deduced from Figure 3, the gradient of the liquid phase velocity is very high near the wall, which leads to a high shear rate near the wall and consequently, break-up is enhanced in this region. A more quantitative comparison is provided in Figure 4, in which the predicted bubble size is compared with the experimental measurements in the radial direction and here, numerical results fit well with the experimental data. Base on the Figures 1 to 4, it can be concluded that $S_\gamma$ model in STAR-CD is capable of reasonably and accurately predicting the bubble size and bubble size distribution in gas-liquid bubbly flows.

It should be pointed out that in all our simulations, good agreement was achieved between the numerical results and the experimental measurements for both phases’ velocity profiles and radial void distributions.

**Figure 1:** Predicted Sauter mean diameter contour from $S_\gamma$ model in STAR-CD.

**Figure 2:** Comparison of the simulated voidage radial distribution with the corresponding experimental data.
The sensitivity of the $S_\gamma$ model to the moment $\gamma$

As described in the Section 3, $S_\gamma$ is a volumetric conserved $\gamma$-moment distribution. $S_0$ represents the bubble number density, $n$, and $S_2$ is the interfacial area density. In principal, bubble size, $d_B$ and/or interfacial area density, $a_i$, could be obtained from the zero-th-moment distribution ($S_0$) as well as those obtained from the second-moment distribution, $S_2$. Figure 5 displays the comparison of the bubble size predicted by both moments. It is seen in Figure 5 that, the bubble size distribution predicted by the $S_0$ model slightly differs that obtained from $S_2$ model, which is also found in the comparison of the interfacial area density, $a_i$, as illustrated in Figure 6. Though the difference is very small, it seems that $S_0$ model produces a better solution for the bubble size while $S_2$ provides a better solution for the interfacial area density. Nevertheless, Figures 5 and 6 suggest that $S_\gamma$ model is compatible for all moments in practice.

The sensitivity of the $S_\gamma$ model to the drainage mode

The drainage model provides the drainage time during the viscous collision. Currently, the selection of the drainage mode bases more on experience than theoretical. Drainage modes 2, 3 and 4 are valid for bubbles and droplets, which qualitatively provides the mobility of the particle (bubble/droplet) surface. When the liquid is not contaminated, the bubble surface is clear and is fully mobile (drainage mode 4); when the liquid is slightly contaminated, the mobility of the bubble surface is reduced (drainage mode 3), and when the liquid is highly contaminated, there are small particles aggregating on the bubble surface, so the mobility of the bubble surface is greatly reduced (drainage mode 2). With the help of test cases 1, 3 and 4, the effect of the drainage mode on the $S_2$ model is investigated. As found in Eq. 40, the drainage time $t_d$ determines the viscous collision probability, the drainage time $t_d$, $t_d < t_d$, therefore, the predicted bubble size with drainage mode 4 is bigger than those obtained from mode 2 and 3 as seen in Figure 7. As the interfacial area density is proportional to $1/d_B$, hence, in Figure 8, drainage mode 4 under-predicts the interfacial area density. Base on Figs. 7 and 8, it could be concluded that for the Hibiki test cases, drainage
mode 3 produces a better solution. So in the following simulations, drainage mode 3 is adopted.

\[ j_G = 0.321 \text{ m/s}, \quad j_L = 0.986 \text{ m/s}, \quad z/D = 53.5 \]

Figure 7: Comparison of the simulated interfacial area density with the corresponding experimental data. Different drainage modes were studied here.

\[ a_i = 6\alpha_G/d_B \]

Figure 8: Comparison of the predicted interfacial area density with the corresponding experimental data. Different drainage modes were studied here.

The performance of the \( S_\gamma \) model

Figure 9 exhibits the comparison of the radial bubble size distributions obtained from the numerical simulation with the \( S_\gamma \) model and the experiment measurements. It is seen here that, first of all, the trend of the predicted bubble size distribution agrees with the experimental profile. The difference appears near the wall, where the numerical results under-predict the measurements for both cases. Probably, this is because that the \( S_\gamma \) model gives a stronger breakup rate and relatively weaker coalescence in the near wall region. Meanwhile, the complex local fluid structure and turbulent intensity also affect the breakup and coalescence models, especially in case the gas and liquid superficial velocities are higher. It is also easily observed in this figure that the increase of the gas phase superficial velocity directly leads to an increase of the bubble size in the core region. This is due to the fact that higher gas phase superficial velocity leads to a higher gas phase volume fraction and this in turn increases the coalescence. It should be pointed out that, in order to achieve a good agreement between the numerical results and the experiment measurements, different calibration coefficient \( F_{cl} \) in Eq.37 are used: case 1, \( 3 \times 10^{-3} \), case 5, \( 5 \times 10^{-3} \) and case 6, \( 3 \times 10^{-2} \). This in turn shows that the current model constants are valid for the dilute systems, volume fraction correction should be accounted for in the near future.

\[ j_G = 0.471 \text{ m/s}, \quad j_L = 2.01 \text{ m/s}, \quad z/D = 53.5 \]

Figure 9: Comparison of the simulated Sauter mean diameter radial distribution with the corresponding experimental data.

Figure 10 shows the interfacial area density radial distributions obtained from simulation and experiment measurements. Similar to the finding in the comparison of the bubble size distribution, difference between the numerical results and measurements lies in the near wall region. Main reason is the under-prediction of the bubble size in the near wall region as the interfacial area density is related to the bubble size \( a_i = 6\alpha_G/d_B \). With an under-prediction of the bubble size in cases 5 and 6, the interfacial area density, \( a_i \) near the wall is over-predicted. It is further found here, that with the increase of the gas phase superficial velocity, the interfacial area density does not dramatically change as found in the gas phase volume fraction. The main reason is that though there is an increase of the voidage, the bigger bubbles greatly reduce the interfacial area density, which leads to the results that the shape of the interfacial area density radial distribution differs to that of the void fraction.
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Figure 10: Comparison of the simulated and experimental profiles of the interfacial area density.

CONCLUSION

Numerical simulations of the gas-liquid two-phase flow with high superficial velocity in a vertical pipe were conducted with the use of the commercial software package STAR-CD 3.27. The applicability and performance of the $S_\gamma$ model in Eulerian modelling of gas-liquid bubbly flow were studied. The sensitivity of the $S_\gamma$ model to the distribution moment, $\gamma$, and the drainage mode were also investigated. The numerical results were compared with the available experimental data of Hibiki et al., (2001). Good agreement was achieved for the phase axial velocity and radial void fraction for all tested cases. It is found in this work that the second-moment $S_\gamma$ model in STAR-CD is capable of reasonably predicting bubble size and its distribution even in high void fraction. Except in the near wall region, simulated bubble size and therefore the interfacial area density fit well with the experiment measurements. This can be attributed to the fact that in the near wall region, the liquid phase shear rate is very high, which leads to a breakup rate that is too high in the current model.

It is observed that the predicted bubble size and interfacial area density obtained from $S_0$ model and $S_\gamma$ model are more or less the same, which indicates that the numerical results is independent of the distribution moment, $\gamma$. It is further found that, the drainage mode greatly affects the bubble size: the increase of the mobility of the bubble surface enhances the coalescence in the current model and leads to an over-prediction of the bubble size in the pipe core. The bubble size increases with the increase of the gas phase superficial velocity while the interfacial area density varies less as the interfacial area density is a combined function of the bubble size and local gas hold-up. Due to the complexity of the breakup and coalescence phenomena, further effort to develop and understand the mechanisms and advanced models of the breakup and coalescence are still needed. Further work to improve the performance of the $S_\gamma$ model is needed to obtain a good solution for the bubble size in the near wall region. Also, transient simulations are under consideration.

REFERENCES


